Approximate Optimization for Proportional Fair AP Association in Multi-rate WLANs*

Wei Li[1], Yong Cui[2], Shengling Wang[2], and Xiuzhen Cheng[3]

[1] Beijing University of Posts and Telecommunications, Beijing, P.R.China
liwei04213@126.com
[2] Tsinghua University, Beijing, P.R.China
{cy,slwang}@csnet1.cs.tsinghua.edu.cn
[3] The George Washington University, Washington DC, USA
cheng@gwu.edu

Abstract. In this study, we investigate the problem of achieving proportional fairness via Access Point (AP) association in multi-rate WLANs. This problem is formulated as a non-linear program with an objective function of maximizing the total user bandwidth utilities in the whole network. It is NP-hard, and therefore effort in this paper is made to seek approximate solutions. We propose a centralized algorithm to derive the user-AP association via relaxation. Such a relaxation may cause a large integrality gap. Therefore a compensation function is introduced to guarantee that our algorithm can achieve at least half of the optimal solution in the worst-case scenario theoretically. Extensive simulation study has been reported to validate and compare the performances of our algorithms with those of the state-of-the-art.

Keywords: AP association, bandwidth allocation, multi-rate WLANs, proportional fairness.

1 Introduction

By default, each user in IEEE 802.11 WLANs associates with the AP that has the largest received signal strength indicator (RSSI). As typically users are not uniformly distributed among all APs, RSSI based approach may overload some APs while leave others carrying very light load or even being idle. This load unbalancing could result in unfair bandwidth allocation. Although the network is supposed to serve fairly at high performance, fairness and efficiency are often in conflict with each other. With the development of multi-rate WLANs, this problem has become even more challenging, as users with different bit rates intend to share the same WLAN.

802.11 MAC protocol provides equal long-term transmission opportunities to users. Therefore, users with the same frame size can achieve equal throughput (i.e. throughput-based fairness). However, in multi-rate WLANs, throughput-based fairness requires that users with lower bit rates occupy the channel for a longer time

* Supported by National Natural Science Foundation of China (no.60873252), National Major Basic Research Program of China (no.2009CB320503), and the US National Science Foundation (CNS-0831852).

G. Pandurangan et al. (Eds.): WASA 2010, LNCS 6221, pp. 36–46, 2010.

than those with higher bit rates, drastically reducing the network throughput [1]. To overcome this problem, time-based fairness is proposed such that each user can obtain an equal share of channel occupancy time. Recent research [1] has shown that time-based fairness outperforms throughput-based fairness in multi-rate WLANs.

There are two fairness criteria that are widely used in network resource assignment. *Max-min fairness* [2] distributes resources as equally as possible among users. While *proportional fairness* [3], on the other hand, allocates bandwidth to users in proportion to their bit rates to maximize the sum of the bandwidth utility of all users. Proportional fairness has been utilized to effectively exploit the tradeoff between fairness and network performance [3], [4].

Fairness, load balancing, and AP selection are three interrelated dimensions of the resource management problem. Nevertheless, very few recent researches jointly consider these three dimensions in multi-rate WLANs. Though different algorithms have been proposed to achieve fairness [1], [5], they only consider the problem at a single AP instead of the whole network. Other approaches optimize the AP association for efficient resource assignment [6], but they do not consider fairness. Load balancing has been proposed to optimize the resource assignment in [7], but fairness again is not considered.

It has been observed that jointly considering AP association and fairness can effectively improve the aggregated network throughput [4], [8] in WLANs, since users may reside in the overlapping coverage areas of multiple APs while each user is only associated with one AP at a time. Li *et al.* [4] propose two approximate AP selection schemes, cvapPF and nlapPF, for periodic offline optimization. Both cvapPF and nlapPF rely on relaxation and rounding to obtain an integral user-AP association. Bejerano *et al.* [8] demonstrate the strong correlation between fairness and load balancing, and propose a load balancing technique to obtain an optimal max-min fair bandwidth allocation.

In this paper, we propose an algorithm for AP selection to achieve proportional fairness. In our system model, the resources at all APs are considered as a whole when allocating bandwidth fairly to users. With this network-wide fairness objective, load balancing is automatically considered. Our Non-linear Approximate Optimization for Proportional Fairness (NLAO-PF) is centralized and can be adopted periodically in real multi-rate WLANs applications. Since the non-linear optimization problem is NP-hard [4], NLAO-PF is decomposed into four steps to simplify the issue and improve the degree of approximation. Our problem formulation is motivated by the non-linear program in [4] but we adopt a completely different approach to relax the variables in our approximation algorithm design. By introducing a compensation function to the objective function to narrow down the gap caused by relaxation, the total utility of the bandwidth allocation via NLAO-PF is proved to be at least 1/2 of the optimal in the worst case. Our comparison-based simulation study indicates that NLAO-PF performs well when the users are distributed randomly and uniformly in the network. Moreover, the performance is even better when users are distributed in a hot spot area.

The rest of the paper is organized as follows. Our system model is introduced in Section 2. The algorithm is detailed in Sections 3. After presenting the evaluation results in Section 4, we conclude this paper in Section 5.

2 System Model

Our network topology models an IEEE 802.11 based multi-rate WLAN network that consists of multiple APs. Each AP has the same limited coverage area and serves users in its area. Overlapping coverage areas of adjacent APs may exist. The union of the coverage areas of all APs forms the network coverage area. We assume that each AP transmits messages with the same power as defined by IEEE 802.11. We further assume that each user is covered by at least one AP, and each AP has at least one associated user. The notations and definitions to be utilized are summarized in Table 1.

Table 1. Notations

Symbol	Semantics
A	The set of all access points (AP).
A_i	The set of APs that can be associated with (cover) user *i*.
n	$n=\|A\|$, the number of APs.
U	The set of all users
m	$m=\|U\|$, the number of users.
γ_{ij}	The SINR of user *i* when associated with AP *j*.
g_{ij}	The channel gain from AP *j* to user *i*.
p_j	The transmission power of AP *j*.
N_0	The receiver noise power.
w_i	The weight of user *i*.
b_i	The effective bandwidth allocated to user *i*.
r_{ij}	The effective bit rate between user *i* and AP *j*.
x_{ij}	The association coefficient between user *i* and AP *j*.
X	The 0-1 user-AP association matrix.
t_{ij}	The transmission time between user *i* and AP *j*.
T	The transmission time allocation matrix.

Table 2. The Relationship between SINRs and Rates

γ_{ij} (dB)	6-7.8	7.8-9	9-10.8	10.8-17	17-18.8	18.8-24	24-24.6	24.6-
r_{ij} (Mbps)	6	9	12	18	24	36	48	54

As we have known, a user in an overlapping coverage area will be interfered by other APs. The effective bit rate of a user in an 802.11 network is determined by the experienced SINR of the user. More precisely, let γ_{ij} denote the SINR of user *i* when associated with AP *j*. We have,

$$\gamma_{ij} = \frac{g_{ij} p_j}{\sum_{k \in A_i \cap k \neq j} g_{ik} p_k + N_0}. \quad (1)$$

Further, the relationship between the effective bit rates and the SINR ranges in the 802.11 network is shown in Table 2 [9].

It is assumed that the network is saturated such that all APs are busy all the time and all users always have data ready to send. We will consider *a unit of time* in which the network is stable, with no new user joins and no current user leaves. This means that under our consideration the total transmission time of an AP is equal to 1. Each AP assigns transmission times to users in accordance with proportional fairness. A user is allowed to choose only one AP within the unit time.

We formulate the AP association problem based on proportional fairness to a non-linear program. In our system model, the resources at all APs are considered as a whole when allocating bandwidth to users. With this network-wide fairness objective, load balancing is automatically taken into account. We intend to maximize the *total utility of the user bandwidth*, which is defined to be *the sum of logarithms of the bandwidths allocated to all users* [3]. Since the effective bandwidth of user *i* is $b_i = \sum_{j=1}^{n} x_{ij} t_{ij} r_{ij}$, we obtain the following optimization formulation:

$$\begin{aligned} \max \quad & \sum_{i=1}^{m} w_i \log\left(\sum_{j=1}^{n} x_{ij} t_{ij} r_{ij}\right) & & \\ s.t. \quad & 1 \le i \le m, \sum_{j=1}^{n} x_{ij} = 1\,, & \text{(a)} & \quad (2) \\ & 1 \le j \le n, \sum_{i=1}^{m} x_{ij} t_{ij} = 1\,, & \text{(b)} & \\ & 1 \le i \le m, 1 \le j \le n, x_{ij} \in \{0,1\}\,, & \text{(c)} & \\ & 1 \le i \le m, 1 \le j \le n, t_{ij} \in [0,1]\,. & \text{(d)} & \end{aligned}$$

Eq. (2) is referred as a Non-linear Program (***NLP***). Note that our objective function considers the weights of users, which reflects their priorities in a real network. The constraint (a) indicates that each user can associate with one and only one AP; the constraint (b) requires that the total transmission time of each AP *j* is equal to 1; the constraint (c) assures that x_{ij} is a binary variable that is equal to 1 if and only if user *i* associates to AP *j*; and the constraint (d) specifies the range of the variable t_{ij}. We can prove that ***NLP*** is NP-hard by slightly adapting the reduction procedure proposed in [4]. Note that this problem formulation is motivated by [4] but our approach to solving the problem via relaxation, as elaborated in Section 3, is fundamentally different and totally novel.

3 The NLAO-PF Algorithm

Since ***NLP*** is NP-hard, we propose an approximation algorithm Non-Linear Approximate Optimization for Proportional Fairness (NLAO-PF) outlined in Alg.1, to simplify the issue and improve the degree of approximation.

Algorithm 1. NLAO-PF

1. $\{t'_{ij}\}$= solve ***r-NLP*** $\{\{w_i\}, \{r_{ij}\}\}$.
2. Get fractional solution $\{x'_{ij}\}$= solve ***c-NLP*** $\{\{w_i\}, \{r_{ij}\}, \{t'_{ij}\}\}$.
3. Get integral solution $\{x_{ij}\}$ by a rounding process $\{\{t'_{ij}\}, \{x'_{ij}\}\}$.
4. Redistribute transmission time to obtain $\{t_{ij}\}$ and calculate $\{b_i\}$.

The basic idea of NLAO-PF is to relax the binary variable x_{ij} such that each user is allowed to associate with multiple APs within a unit time. This relaxation may result in a large integrality gap [10]. To overcome this problem, we modify the objective function of ***NLP*** by adding a compensation function $g(X,T)$ in NLAO-PF, which is defined as follows.

Definition 1. *The compensation of user i on AP j* is defined by $w_i x_{ij} t_{ij} \log(r_{ij})$, if $r_{ij}>0$; 0, otherwise. Thus the compensation of user *i* to all APs can be expressed by $w_i(\sum_{j=1}^{n} x_{ij} t_{ij} \log(r_{ij}))$. Therefore *the compensation function* $g(X,T)$ can be defined correspondingly as follows:

$$g(X,T)=\sum_{i=1}^{m} w_i (\sum_{j=1}^{n} x_{ij} t_{ij} \log(r_{ij})) . \quad (3)$$

This compensation function is introduced to improve the lower bound of our algorithm to effectively narrow down the integrality gap caused by relaxation. The steps of NLAO-PF are detailed in the following subsections.

3.1 Relaxed Optimization Program

The first step of NLAO-PF is to solve the following relaxed optimization problem to obtain an optimal $\{t'_{ij}\}$.

$$\begin{aligned} \max \quad & \sum_{i=1}^{m} w_i \log(\sum_{j=1}^{n} t'_{ij} r_{ij}) + \sum_{i=1}^{m} w_i (\sum_{j=1}^{n} t'_{ij} \log(r_{ij})) \\ s.t. \quad & 1 \le i \le m, \sum_{j=1}^{n} t'_{ij} \le 1, && \text{(a)} \\ & 1 \le j \le n, \sum_{i=1}^{m} t'_{ij} = 1, && \text{(b)} \\ & 1 \le i \le m, 1 \le j \le n, t'_{ij} \in [0,1]. && \text{(c)} \end{aligned} \quad (4)$$

Eq. (4) is referred as the *relaxed Non-linear Program* (***r-NLP***). Compared with Eq. (2), ***r-NLP*** replaces t_{ij} by t'_{ij}, sets x_{ij} =1, and includes the compensation function in its objective function. The constraint (a) indicates that the total transmission time of user i with all APs cannot surpass 1; the constraint (b) requires that the total transmission time of each AP is equal to 1, which means that all APs are saturated in the unit time; and the constraint (c) defines the range of the variable t'_{ij}. Obviously, the optimal solution for $\{t'_{ij}\}$ from Eq. (4) can be found in polynomial time [4].

3.2 Fractional Association

After solving ***r-NLP***, we obtain the transmission time $\{t'_{ij}\}$. Now we take $\{t'_{ij}\}$ as the input, and get the fractional user-AP association $\{x'_{ij}\}$. Because of the requirements for solving convex programs, we change the linear equality constraint of ***NLP*** to a linear inequality constraint in the following problem formulation, which does not change the solution value.

$$\begin{aligned} \max \quad & \sum_{i=1}^{m} w_i \log(\sum_{j=1}^{n} x'_{ij} t'_{ij} r_{ij}) + \sum_{i=1}^{m} w_i (\sum_{j=1}^{n} x'_{ij} t'_{ij} \log(r_{ij})) \\ s.t. \quad & 1 \le i \le m, \sum_{j=1}^{n} x'_{ij} > 0, && \text{(a)} \\ & 1 \le j \le n, \sum_{i=1}^{m} x'_{ij} t'_{ij} = 1, && \text{(b)} \\ & 1 \le i \le m, 1 \le j \le n, x'_{ij} \ge 0. && \text{(c)} \end{aligned} \quad (5)$$

Eq. (5) is referred as the *complemented Non-linear Program* (***c-NLP***). Its objective function is designed to approximate the optimal solution to ***NLP***. The constraint (a) indicates that a user should connect with at least one AP; the constraint (b) forces the total transmission time of each AP be equal to 1; and the constraint (c) defines the range of x'_{ij} for the case of fractional association. Note that here we take $\{t'_{ij}\}$ obtained from Eq. (4) as the input to ***c-NLP*** and obtain the optimal association $\{x'_{ij}\}$ for ***c-NLP*** given $\{t'_{ij}\}$.

We can prove that the gap introduced by our relaxation procedure is bounded. Let $f(X,T)=\sum_{i=1}^{m} w_i \log(\sum_{j=1}^{n} x_{ij} t_{ij} r_{ij})$, $h(X,T)=f(X,T)+g(X,T)$. Then the objective functions of ***r-NLP*** and ***c-NLP*** become $h(X=1, T)$ and $h(X,T)$, respectively. Correspondingly, $h(X=1, T')$ and $h(X',T')$ are the solutions obtained from ***r-NLP*** and ***c-NLP***, respectively.

Theorem 1. Let $f(X^*,T^*)$ be the optimal solution to ***NLP***. Then $f(X^*,T^*) \le h(X',T') \le 2 f(X^*,T^*)$.

Proof. With $\sum_{i=1}^{m} x'_{ij} t'_{ij} = 1$ (constraint (b) in Eq. (5)) and $r_{ij} \ge 1$, $f(X',T') \ge g(X',T') \ge 0$. Thus $h(X',T') \le 2f(X',T') \le 2f(X^*,T^*)$. Note that $f(X^*,T^*)$ is also feasible to ***c-NLP*** from the relationship between ***NLP*** and ***c-NLP***. Thus we have, $f(X^*,T^*) \le f(X=1,T')+g(X=1,T')=h(X=1,T') \le h(X',T')$, where the last inequality holds true from the fact that $h(X=1, T')$ is feasible to ***c-NLP***. ■

3.3 Optimization Program Rounding

In this step, we use the rounding algorithm proposed in [11] to obtain an integral association matrix X. That is, we fix the time allocation $\{t'_{ij}\}$ and replace the fractional association $\{x'_{ij}\}$ by a 0-1 variable $\{x_{ij}\}$ that encodes the desired association of users to APs. The description of the rounding scheme is as follows.

First, we construct a bipartite graph $G(x)=(U,V,E)$, where the set U represents the users in the network, and the set V consists of AP nodes denoted by $V=\{v_{jk}: j=1,\ldots,n, k=1,\ldots,Q_j\}$, with $Q_j = \lceil \sum_{i=1}^{m} x_{ij} \rceil$. This means that each AP may have multiple nodes in V. The edges in $G(x)$ are constructed in the following way. If $Q_j \le 1$, there is only one node v_{j1} corresponding to AP j. For each $x'_{ij}>0$, add an edge $e(u_i, v_{j1})$ to E, and set $x'(u_i, v_{j1})=x'_{ij}$, where $x'(e)$ is the fractional association weight of the corresponding user and AP. Otherwise, find the minimum index i_k such that $\sum_{i=1}^{i_k} x'_{ij} \ge k$. For $i=i_{k-1}+1,\ldots, i_k-1$ and $x'_{ij}>0$, add edges $e(u_i, v_{jk})$ and set $x'(u_i, v_{jk})=x'_{ij}$. For $i=i_k$, add the edge $e(u_i, v_{jk})$ and set $x'(u_i,v_{jk})=1-\sum_{i=i_{k-1}+1}^{i_k-1} x'(u_i,v_{jk})$. If $\sum_{i=1}^{i_k} x'_{ij} > k$, add the edge $e(u_i, v_{j(k+1)})$ and set $x'(u_i,v_{j(k+1)})=\sum_{i=1}^{i_k} x'(u_i,v_{jk})-k$. The *profit* of each edge $e(u_i, v_{jk})$ in E is defined to be $w_i \log(t'_{ij} r_{ij})$.

Second, we find a maximum-profit matching $M(x)$ that matches each user node to an AP node in $G(x)$. For each edge $e(u_i, v_{jk})$ in $M(x)$, schedule user i on AP j and set $x_{ij}=1$. Set other x_{ij}'s to be 0. Since the fractional association $\{x'_{ij}\}$ specifies a fractional matching, such a maximal matching does exist and it determines the integral association $\{x_{ij}\}$. More details can be found in [11].

Note that $\{t'_{ij}\}$ and $\{x'_{ij}\}$ are computed from ***r-NLP*** and ***c-NLP***, respectively. The rounding scheme constructs an integral assignment $\{x_{ij}\}$. We denote this integral solution as $f(X^a,T')$, which is also feasible to ***NLP***. We have

Theorem 2. $f(X^a,T') \geq f(X^*,T^*)/2$.

Proof. Note that $\{x_{ij}\}$ is obtained by employing the rounding scheme proposed by Shmoys and Tardos in [11], which proves the following property: $f(X^a,T') \geq f(X',T')$. Thus, $f(X^a,T') \geq f(X',T') \geq [f(X',T')+g(X',T')]/2 = h(X',T')/2 \geq f(X^*,T^*)/2$, where the last inequality holds from Theorem 1. ■

3.4 Transmission Time Redistribution

Since the user-AP association changes after rounding, we need to redistribute the transmission time. This is the last step of NLAO-PF, in which we assign transmission times to users according to proportional fairness.

Theorem 3. Let $\{x_{ij}\}$ be the integral user-AP association coefficients obtained from the rounding procedure outlined in Section 3.3. Given $\{x_{ij}\}$, the unique optimal transmission time assigned to user i by AP j according to proportional fairness is $t_{ij} = x_{ij} w_i \big/ (\sum_{k=1}^{m} x_{kj} w_k)$.

Proof. (a) First, we consider the case of a single AP. Assume that the number of users covered by the AP is m. Since the objective function of Eq. (2) is the sum of logarithms, maximizing the total utility of the user bandwidth (Eq. (2)) is equivalent to maximizing Eq. (6):

$$\prod_{i=1}^{m} (t_{ij} r_{ij})^{w_i} = \prod_{i=1}^{m} (t_{i1} r_{i1})^{w_i} = \prod_{i=1}^{m} (t_{i1})^{w_i} \prod_{i=1}^{m} (r_{i1})^{w_i} . \tag{6}$$

Note that $\{r_{i1}\}$ is the set of optimization constant. Therefore maximizing Eq. (6) is equivalent to maximizing Eq. (7):

$$\prod_{i=1}^{m} (t_{i1})^{w_i} = (\underbrace{t_{11} t_{11} \cdots t_{11}}_{w_1})(\underbrace{t_{21} t_{21} \cdots t_{21}}_{w_2}) \cdots (\underbrace{t_{m1} t_{m1} \cdots t_{m1}}_{w_m}) . \tag{7}$$

Since $\sum_{i=1}^{m} t_{i1} = 1$, Eq. (7) is maximized if and only if if $t_{11}/w_1 = t_{21}/w_2 = \cdots = t_{m1}/w_m = 1\big/(\sum_{i=1}^{m} w_i)$. Thus we have $t_{i1} = w_i \big/ (\sum_{k=1}^{m} w_k)$.

(b) Now we consider the case of multiple APs. Let x_{kj} be a 0-1 variable denoting the association coefficient between user k and AP j. Then $\sum_{k=1}^{m} x_{kj} w_k$ is the sum of the weights of all users associated to AP j. With a similar analysis as that of case (a), the optimal transmission time given $\{x_{ij}\}$ can be calculated by Eq. (8).

$$t_{ij} = x_{ij} w_i \big/ (\sum_{k=1}^{m} x_{kj} w_k) . \tag{8}$$

We conclude that given $\{x_{ij}\}$, our transmission time assignment based on proportional fairness is unique and optimal. ■

The solution obtained from our algorithm NLAO-PF can be denoted as $f(X^a,T^a)$. Based on Theorems 2 and 3, we have $f(X^a,T^a) \geq f(X^a,T') \geq f(X^*,T^*)/2$. That is, the approximate solution obtained from NLAO-PF is no less than half of the optimal solution of ***NLP***.

4 Evaluation

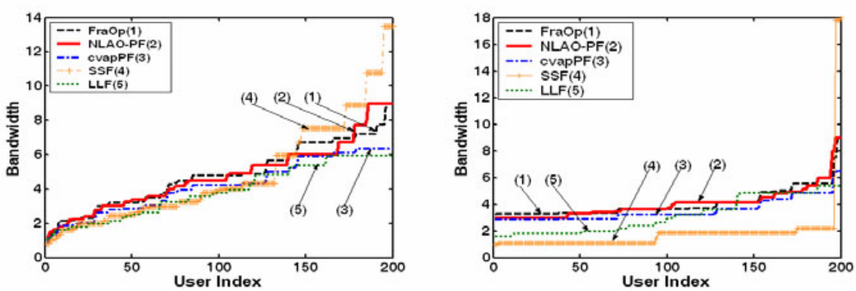
For ease of comparison, we employ the same simulation settings as those in [4], which are detailed as follows. We place a total of 20 APs on a 5 by 4 grid, with each AP on a grid point. The coverage area of each AP is set to 150 meters and the distance between two adjacent APs is set to 100 meters. We arrange 50~300 users to simulate different levels of network loads. Assume that the transmission power of each AP is 20dBm [12]. The coverage area of the network is the union of the coverage areas of all APs. Assume that all users have the same weight. There are two types of user distributions under our consideration: (1) users are randomly and uniformly distributed within the coverage area of the network; (2) users are randomly positioned in a circle-shaped hotspot with a radius of 100 meters near the center of the 20-AP network.

We employ a simple wireless channel model in which the effective bit rate only depends on the experienced SINR. For simplicity, we adopt the values commonly advertised by 802.11a which is shown in Table 2. The channel gain is modeled by the following equation,

$$g_{ij}=s_{ij}d_{ij}^{-4}\ , \qquad (10)$$

Table 3. Data Analysis on Different Algorithms

Case	Algorithm	Ave.(Mbps)	Var.(Mbps)	Fairness	Total utility
Uniform	FraOp	4.82	3.42	0.87	129.18
	NLAO-PF	***4.79***	***3.78***	***0.86***	***128.01***
	cvapPF	4.19	3.75	0.87	117.32
	SSF	4.82	9.12	0.71	120.18
	LLF	3.87	3.36	0.85	109.53
Hotspot	FraOp	4.06	1.07	0.94	119.51
	NLAO-PF	***4.05***	***1.23***	***0.93***	***118.73***
	cvapPF	3.55	1.53	0.93	108.01
	SSF	2.04	14.29	0.39	38.12
	LLF	3.17	1.93	0.85	92.64



(a) uniform case (b) hotspot case

Fig. 1. The user bandwidth of different AP association algorithms

where s_{ij} is a log-normally distributed shadowing factor, and d_{ij} is the distance between user i and AP j. Shadowing factors are generated according to the Viterbi model [13], with $E(s_{ij})$=0dB and $\sigma(s_{ij})$=10dB. The receiver noise power N_0 is set to -80dBm.

Fairness is quantified by the Jain's fairness index [14], which is defined as follows,

$$J = \left(\sum_{i=1}^{m} b_i^2\right) \Big/ \left(\sum_{i=1}^{m} b_i\right)^2 , \quad (11)$$

where larger values of $J \in [0,1]$ indicate a better fairness. In proportional fairness, larger values of the total utility, which reflects the tradeoff between the aggregated throughput and fairness, indicate a better performance.

During our simulation, the network topology structure and the effective bit rates are assumed to be steady. There is no backhaul capacity limitation.

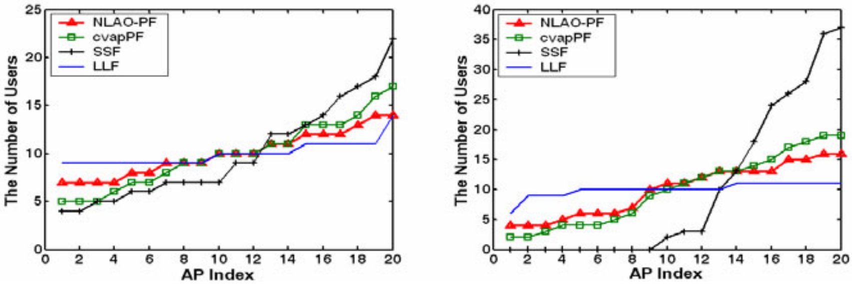
We compare the performances of the following algorithms: (1) NLAO-PF; (2) cvapPF [4]; (3) Strongest Signal First (SSF); (4) Least Load First (LLF) [8]; (5) Since the problem of AP association based on proportional fairness is NP-hard, NLAO-PF and cvapPF only obtain approximate solutions. For comparison purpose, we use the result obtained from ***r-NLP*** without the compensation function g(X,T) as a benchmark and call it FraOp.

We have performed extensive simulations by varying the number of users and obtained qualitatively similar results. Thus in this subsection we only report the results for the 200-user case. The statistics of achieved bandwidth, Jain's fairness index, and total bandwidth utility of different algorithms are presented in Table 3.

First Fig.1 plots the achieved per-user bandwidth in Mbps *vs.* user index, with the users sorted by their bandwidths in increasing order. In the uniform case, SSF achieves a little bit higher average bandwidth, demonstrating a much larger variance in the bandwidth allocation and a poorer fairness. However, in the hotspot case where users reside in the vicinity of certain APs, leading to a more intensive competition for resources and a more imbalanced network load. Obviously, SSF aggravates the extent of load imbalance and enlarges the bandwidth allocation variances without considering fairness. It is also difficult for LLF to enhance the user bandwidth effectively because it only takes into account load balancing but ignores the user rate. On the other hand, in these two cases, NLAO-PF and cvapPF can both achieve proportional fairness, but NLAO-PF outperforms cvapPF in terms of bandwidth allocation with a value closer to FraOp. The fairness index of these two algorithms is almost the same. Moreover, the total utility of NLAO-PF is 99% of that of FraOp, while that of cvapPF is about 90%. Therefore, we conclude that NLAO-PF outperforms cvapPF, SSF and LLF.

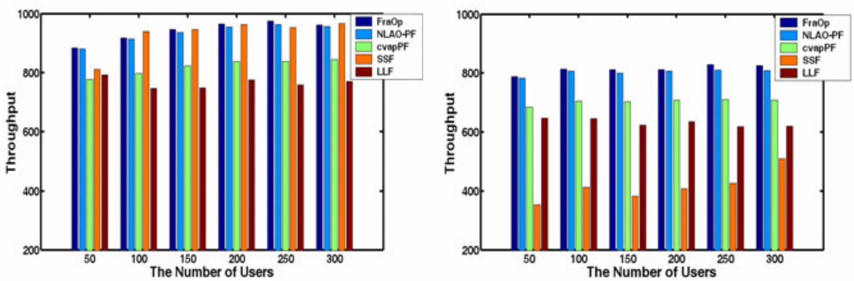
Second, the number of users *vs.* AP index is shown in Fig.2, with the APs sorted by their users in increasing order. The number of users associated with an AP is considered as the load metric. In these two cases, both NLAO-PF and LLF perform better than the other two algorithms in terms of load balancing. However, from Table 3, we observe that LLF has a lower average bandwidth and a poorer fairness than NLAO-PF, since it ignores the user rate. It can be concluded that NLAO-PF offers a more effective tradeoff between the bandwidth allocation and load balancing.

Besides per-user bandwidth, we also compare the aggregated throughput of all algorithms. Fig.3 plots the aggregated throughput in Mbps *vs.* the number of users. In the uniform case, although SSF achieves a higher aggregated throughput, it reduces the level of fairness in bandwidth allocation. Among the remaining three algorithms



(a) uniform case (b) hotspot case

Fig. 2. The load of different AP association algorithms with 200 users



(a) uniform case (b) hotspot case

Fig. 3. The aggregated throughput of different AP association algorithms

(except FraOp), NLAO-PF obtains much higher aggregated throughputs than the other two algorithms, especially in the hotspot case. On the other hand, compared with cvapPF, NLAO-PF has a higher approximate degree, and its optimization results are more prominent. What is more, in the hotspot case, advantages of our algorithm are even more significant.

5 Conclusion

The widespread of multi-rate WLAN applications makes the network management more complex and critical. Fairness and AP association are two hot issues. In multi-rate WLANs, some users may get starved if fairness is not carefully considered. In this paper, we investigate how to optimize user-AP association to achieve proportional fairness, and propose a new AP association algorithm termed NLAO-PF for this purpose. Although the problem of AP association for proportional fairness is NP-hard, NLAO-PF obtains a result that is guaranteed to be at least half of the optimal solution via a compensation function. Simulations confirm that our scheme can achieve proportional fairness in bandwidth allocation and enhance the aggregated throughput effectively. Moreover, in the hotspot case, the advantage of our algorithm is even more significant.

References

1. Tan, G., Guttag, J.: Time-based Fairness Improves Performance in Multi-rate WLANs. In: The USENIX Annual Technical Conf., Boston (2004)
2. Bertsekas, D., Gallager, R.: Data Networks, pp. 524–529. Prentice-Hall, Englewood Cliffs (1987)
3. Kelly, F.P.: Charging and rate control for elastic traffic. European Transactions on Telecommunications 8(1), 33–37 (1997)
4. Li, L., Pal, M., Yang, Y.R.: Proportional Fairness in Multi-rate Wireless LANs. In: IEEE INFOCOM, Phoenix, pp. 1004–1012 (2008)
5. Bredel, M., Fidler, M.: Understanding Fairness and its Impact on Quality of Service in IEEE 802.11. In: IEEE INFOCOM, Rio de Janeiro, pp. 1098–1106 (2009)
6. Abusubaih, M., Wolisz, A.: An Optimal Station Association Policy for Multi-Rate IEEE 802.11 Wireless LANs. In: 10th ACM Symposium MSWiM, pp. 117–123 (2007)
7. Rahul, H., Edalat, F., Katabi, D., Sodini, C.: Frequency-Aware Rate Adaptation and MAC Protocols. In: 15th ACM International Conf. MobiCom, Beijing, pp. 193–204 (2009)
8. Bejerano, Y., Han, S.-J., Li, L.E.: Fairness and load balancing in wireless LANs using association control. IEEE/ACM Trans. Networking 15(3), 560–573 (2007)
9. High-speed Physical Layer in the 5 GHz Band, IEEE Standard 802.11a (1999)
10. Azar, Y., Epstein, A.: Convex Programming for Scheduling Unrelated Parallel Machines. In: 30th ACM symposium Theory of computing, Baltimore, pp. 331–337 (2005)
11. Shmoys, D.B., Tardos, E.: An approx algorithm for the generalized assignment problem. Math. Program 62(3), 461–474 (1993)
12. Radio Resource Measurement of Wireless LANs, IEEE Standard 802.11k (2008)
13. Viterbi, A.J.: CDMA: Principles of Spread Spectrum Communication, pp. 185–186. Addison-Wesley, New York (1995)
14. Jain, R., Chiu, D.-M., Hawe, W.R.: A quantitative measure of fairness and discrimination for resource allocation in shared computer system. Digital Equipment, Technical report, DEC-TR-301 (1984)